IVLE: MA1511: Why dxdy=r dr dtheta

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Tue 9/5/2017 10:25 PM

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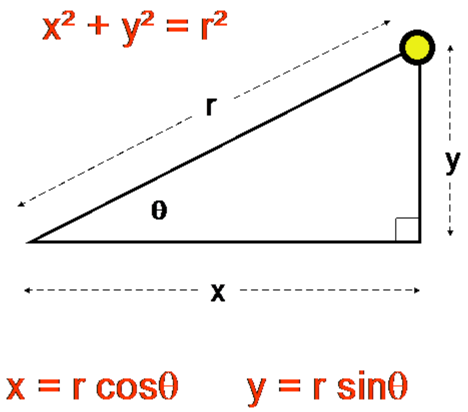
Dear class,

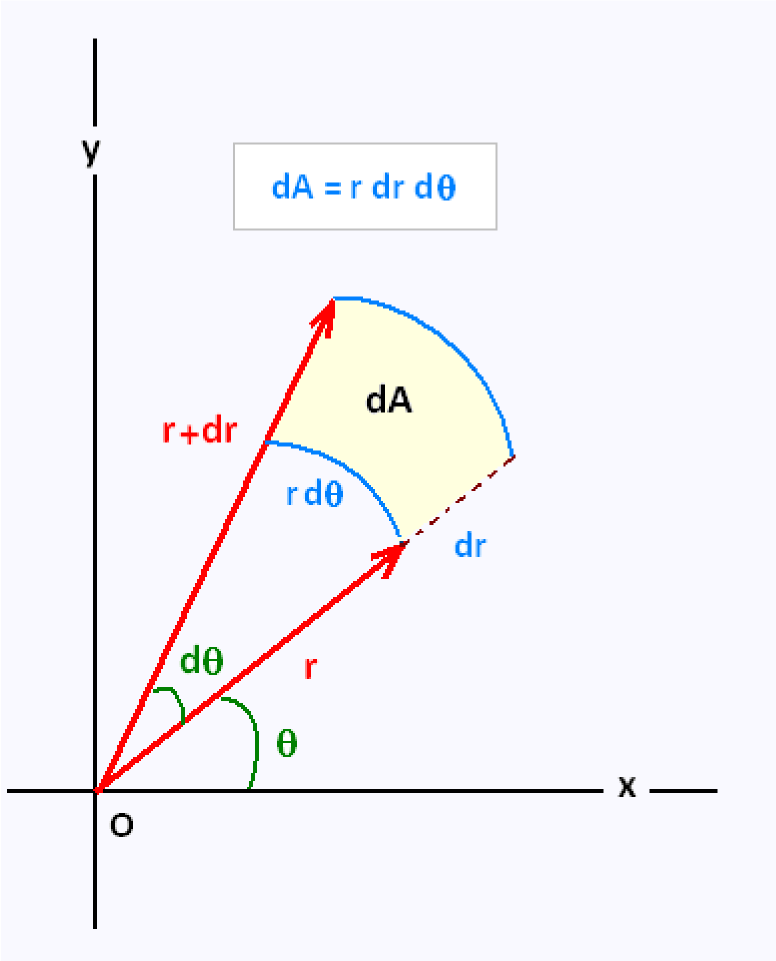
For polar coordinates double integral, we have these 4 important formulae to convert from Cartesian coordinates to Polar Coordinates:

1) $x = r\cos\theta$
2) $y = r\sin\theta$
3) $x^2 + y^2 = r^2$
and
4) $dA = dxdy = r\ drd\theta$

The first 3 are direct consequences of basic trigonometry. (See the diagram below):



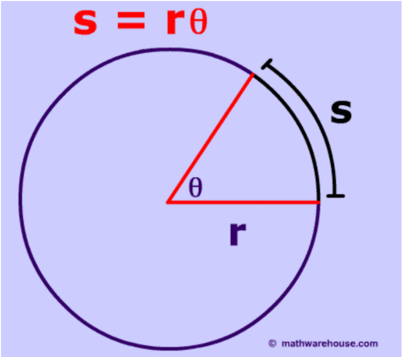
The fourth equation $dA = dxdy = r\ drd\theta$ seems more mysterious, especially the extra "r" that appears. While a rigorous proof is beyond our syllabus, we can show an intuitive "proof":



(Source: https://math.stackexchange.com/questions/1636021/rigorous-proof-that-dx-dy-r-dr-d-theta)

Basically, look at the area element dA, approximated as a rectangle. Its length is dr, while its width is $r \ d\theta$, so we can say that $dA = r \ dr d\theta$. Note that even though dA doesn't look like a rectangle, if it is small enough, it is going to be approximately a rectangle (this is the key idea in Calculus).

Note that the key formula needed here is something learnt in O Level Elementary Math (high school syllabus in Singapore), the arc length formula for a circle:



Best regards,
Chengyuan