Population Equations Summary

Tutor: Wu Chengyuan

This is a summary of the equations needed to solve questions in this topic.

1 Malthus Model

$$\frac{dN}{dt} = BN - DN$$

N: Total population

B: Birth-rate per capita

D: Death-rate per capita

1.1 Exam Questions

2016 Q4(a), 2015 Q3(b)

2 Logistic Equation

$$D = sN \qquad \text{(where } s \text{ is a constant)}$$

$$\frac{dN}{dt} = BN - sN^2$$

$$\widehat{N} = N(0) \qquad \text{(initial population at time } t = 0\text{)}$$

$$N_\infty = B/s \qquad (N_\infty \text{ is the eventual population as } t \to \infty)$$

2.1 Exam Questions

2015 Q4(a)

3 Logistic Case 1: Increasing population ($\widehat{N} < N_\infty$)

$$
\begin{aligned}
N(t) &= \frac{B}{s + (\frac{B}{\widehat{N}} - s)e^{-Bt}} \\
&= \frac{N_\infty}{1 + (\frac{N_\infty}{\widehat{N}} - 1)e^{-Bt}}
\end{aligned}
$$

The second expression can be derived from the first: divide by s in both the numerator and denominator.

3.1 Exam Questions

2016 Q3(b)

4 Logistic Case 2: Decreasing population ($\widehat{N} > N_\infty$)

$$
\begin{aligned}
N(t) &= \frac{B}{s - (s - \frac{B}{\widehat{N}})e^{-Bt}} \\
&= \frac{N_\infty}{1 - (1 - \frac{N_\infty}{\widehat{N}})e^{-Bt}}
\end{aligned}
$$

5 Logistic Case 3: Constant population ($\widehat{N} = N_\infty$)

$N(t) = N_\infty$