

Hyperbolic functions

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For more information, check out https://en.wikipedia.org/wiki/Hyperbolic_function.

1 Definitions

$$\sinh(x) = \frac{1}{2}(e^x - e^{-x})$$

$$\cosh(x) = \frac{1}{2}(e^x + e^{-x})$$

$$\tanh(x) = \frac{\sinh(x)}{\cosh(x)}$$

Note that $\cosh^2 x - \sinh^2 x = 1$.

2 Derivatives

$$\frac{d}{dx} \sinh x = \cosh x$$

$$\frac{d}{dx} \cosh x = \sinh x$$

Derivative of $\tanh x$ can be deduced by quotient rule:

$$\frac{d}{dx} \tanh x = \frac{d \sinh x}{dx \cosh x} = \frac{\cosh^2 x - \sinh^2 x}{\cosh^2 x} = \frac{1}{\cosh^2 x} = \operatorname{sech}^2 x$$

3 Integrals

$$\int \frac{1}{\sqrt{x^2+1}} dx = \sinh^{-1}(x) + C$$
$$\int \sinh(ax+b) dx = \frac{1}{a} \cosh(ax+b) + C$$
$$\int \cosh(ax+b) dx = \frac{1}{a} \sinh(ax+b) + C$$
$$\int \tanh(ax+b) dx = \frac{1}{a} \ln(\cosh(ax+b)) + C$$