Extinction Time Formula

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In this document, we show how to solve Extinction Time (Tutorial 6 Q2) **without** Graphical Calculator or computer.

1 Formula for Extinction Time

$$\begin{aligned} T &= \int_0^T dt = \int_{\widehat{N}}^0 \frac{dN}{N(B-sN)-E} \\ &= \left[\frac{-2}{\sqrt{4sE-B^2}} \tan^{-1}\left(\frac{2sN-B}{\sqrt{4sE-B^2}}\right)\right]_{N=\widehat{N}}^{N=0} \end{aligned}$$

Important: Use **Radian mode** when using this formula.

2 Derivation

$$\begin{aligned} \int \frac{dN}{N(B-sN)-E} &= \int \frac{dN}{NB-sN^2-E} \\ &= -\frac{1}{s}\int \frac{dN}{N^2-\frac{B}{s}N+\frac{E}{s}} \\ &= -\frac{1}{s}\int \frac{dN}{(N-\frac{B}{2s})^2+(\frac{E}{s}-\frac{B^2}{4s^2})} \\ &= -\frac{1}{s}\cdot\frac{1}{\sqrt{\frac{E}{s}-\frac{B^2}{4s^2}}} \tan^{-1}\left(\frac{N-\frac{B}{2s}}{\sqrt{\frac{E}{s}-\frac{B^2}{4s^2}}}\right) \end{aligned}$$

$$= \frac{-2}{\sqrt{4sE - B^2}} \tan^{-1}\left(\frac{2sN - B}{\sqrt{4sE - B^2}}\right).$$

3 Application to Tutorial 6 Q2

In Tutorial 6 Q2, the integral we want to solve is

$$\int_{194600}^{0} \frac{dN}{N(0.09866 - \frac{0.09866N}{194600}) - 10000}.$$

Plugging $\widehat{N} = 194600$, $B = 0.09866$, $s = \frac{0.09866}{194600}$, $E = 10000$ into our formula above, and we can use a normal calculator to get:

$$\begin{aligned} 4sE - B^2 &= 0.0105458 \\ \sqrt{4sE - B^2} &= 0.1026925 \\ 14.906 - (-14.906) &= 29.8 \end{aligned}$$

Answer: 29.8 years.