Vectors

Basic results

- Parallel vectors: $\boldsymbol{a}=\lambda\boldsymbol{b}$
- $\boldsymbol{a},\boldsymbol{b},\boldsymbol{c}$ coplanar: $\boldsymbol{a}=\lambda\boldsymbol{b}+\mu\boldsymbol{c}$
- A, B, C collinear: $\vec{AB}=\lambda\vec{BC}$
- Unit vector $\boxed{\hat{\boldsymbol{b}}=\frac{\boldsymbol{b}}{|\boldsymbol{b}|}}$

Ratio Theorem

$$\boxed{\vec{OC}=\frac{\mu\vec{OA}+\lambda\vec{OB}}{\mu+\lambda}}$$

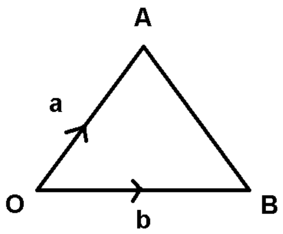
Dot Product

- $\boxed{\boldsymbol{a}\cdot\boldsymbol{b}=|\boldsymbol{a}||\boldsymbol{b}|\cos\theta}$
- $\boxed{\begin{pmatrix}a_1\\a_2\\a_3\end{pmatrix}\cdot\begin{pmatrix}b_1\\b_2\\b_3\end{pmatrix}=a_1b_1+a_2b_2+a_3b_3}$
- $\boxed{\boldsymbol{a}\cdot\boldsymbol{a}=|\boldsymbol{a}|^2}$
- $\boldsymbol{a}\cdot\boldsymbol{b}=0$ if $\boldsymbol{a},\boldsymbol{b}$ are perpendicular
- $\boxed{\boldsymbol{a}\cdot\boldsymbol{b}=\boldsymbol{b}\cdot\boldsymbol{a}}$

Cross Product

- $\boldsymbol{a}\times\boldsymbol{b}=|\boldsymbol{a}||\boldsymbol{b}|\sin\theta\,\hat{\boldsymbol{n}}$, where $\hat{\boldsymbol{n}}$ is the unit vector perpendicular to both **a** & **b**
- If **a, b** are parallel, $\boldsymbol{a}\times\boldsymbol{b}=\boldsymbol{0}$
- $\boxed{\boldsymbol{a}\times\boldsymbol{b}=-\boldsymbol{b}\times\boldsymbol{a}}$

Area (using Cross Product)



Area of $\boxed{\Delta OAB = \frac{1}{2}|\boldsymbol{a} \times \boldsymbol{b}|}$

Equation of Lines

- Vector Eqn: $\boxed{\boldsymbol{r} = \boldsymbol{a} + \lambda \boldsymbol{m}, \lambda \in \mathbb{R}}$
  (**a:** Position vector of point on line, **m**: Direction vector of line)
- Parametric Eqn:
  $x = a_1 + \lambda m_1$
  $y = a_2 + \lambda m_2$
  $z = a_3 + \lambda m_3$
- Cartesian Eqn:
  $\frac{x - a_1}{m_1} = \frac{y - a_2}{m_2} = \frac{z - a_3}{m_3} (= \lambda)$

Length of Projection

Length of projection of **a** on **b** = $\boxed{|\boldsymbol{a} \cdot \hat{\boldsymbol{b}}|}$

Equation of Planes

- Vector Eqn (Parametric): $\boxed{\boldsymbol{r} = \boldsymbol{a} + \lambda \boldsymbol{m_1} + \mu \boldsymbol{m_2}}$
  (**a:** Position vector of point on plane; $\boldsymbol{m_1}, \boldsymbol{m_2}$ are vectors parallel to plane)
- Vector Eqn (Scalar Product): $\boxed{\boldsymbol{r} \cdot \boldsymbol{n} = D}$
  (**n:** vector normal to plane)
- Cartesian Eqn: $ax + by + cz = D$, derived from scalar product form

Distance from Origin to Plane

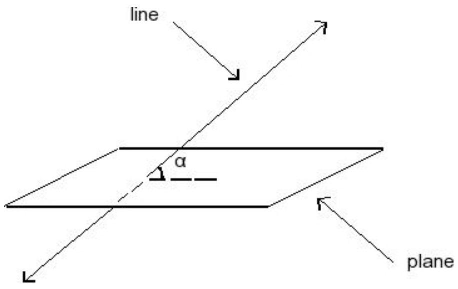
Given a plane $\boldsymbol{r} \cdot \boldsymbol{n} = D$, its distance from the Origin is $\boxed{\frac{D}{|\boldsymbol{n}|}}$.

Angle between 2 Planes

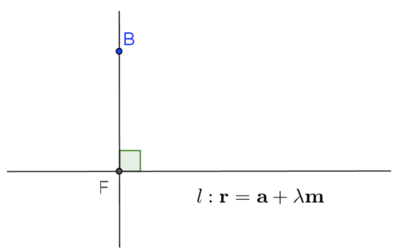
$$\boxed{\cos\theta = \frac{|\boldsymbol{n}_1 \cdot \boldsymbol{n}_2|}{|\boldsymbol{n}_1||\boldsymbol{n}_2|}}$$ ($\boldsymbol{n}_1, \boldsymbol{n}_2$ are the two normal vectors)

Angle between Line and Plane

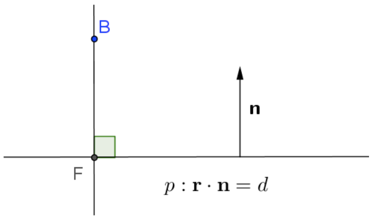
$$\boxed{\sin\theta = \frac{|\boldsymbol{n} \cdot \boldsymbol{m}|}{|\boldsymbol{n}||\boldsymbol{m}|}}$$ (**n** is normal vector of plane; **m** is direction vector of line)



Foot of Perpendicular From Point to Line

	From point (B) to **Line** (l)
(Picture)	
Equation (I): **Where does F lie?**	F lies on the line l . $\overrightarrow{OF} = \boldsymbol{a} + \lambda \boldsymbol{m}$
Equation (II): **Perpendicular**	$\overrightarrow{BF} \cdot \boldsymbol{m} = 0$ $(\overrightarrow{OF} - \overrightarrow{OB}) \cdot \boldsymbol{m} = 0$
Final Step	Substitute Equation (I) into Equation (II) and **solve for** λ .

Foot of Perpendicular From Point to Plane

	From point (B) to **Plane** (p)
(Picture)	
Equation (I): **Where does F lie?**	F lies on the plane p . $\overrightarrow{OF} \cdot \boldsymbol{n} = d$
Equation (II): **Perpendicular**	$\overrightarrow{BF} = k\,\boldsymbol{n}$ $\overrightarrow{OF} - \overrightarrow{OB} = k\,\boldsymbol{n}$ $\overrightarrow{OF} = k\,\boldsymbol{n} + \overrightarrow{OB}$
Final Step	Substitute Equation (II) into Equation (I) and **solve for k**.